

# Scheme for generating entangled states of two field modes in a cavity

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## Abstract

This paper considers a two-level atom interacting with two cavity modes with equal frequencies. Applying a unitary transformation, the system reduces to the analytically solvable Jaynes-Cummings model. For some particular field states, coherent and squeezed states, the transformation between the two bare basis's, related by the unitary transformation, becomes particularly simple. It is shown how to generate, the highly non-classical, entangled coherent states of the two modes, both in the zero and large detuning cases. An advantage with the zero detuning case is that the preparation is deterministic and no atomic measurement is needed. For the large detuning situation a measurement is required, leaving the field in either of two orthogonal entangled coherent states.

## 1 Introduction

Quantum information processing and nonlocality tests rely on the phenomenon of entanglement [1]. This is a purely quantum mechanical correlation between different subsystems, for example, atoms or photons. During the last decades, entanglement has not only been an issue for theoreticians but entangled states have been prepared and measured in various experimental set-ups. Many of the successful demonstrations have been within quantum optical systems [2]. One of the prominent subfields is cavity quantum electrodynamics (cavity QED). In cavity QED, single atomic transitions and single cavity modes can be isolated and coupled coherently, and, for example, preparation of entangled states is achievable [3]. The atom constitute a two-level system, while the cavity mode a harmonic oscillator, and due to the large atom-field coupling together with long lifetimes of the atomic Rydberg states and the use of a high  $Q$  cavity, decoherence effects can often be neglected during the interaction time [4]. Usually the atom is sent through the cavity and interacts resonantly or nearly resonantly with one of the cavity modes. The dynamics is, in most cases, well described by the analytically solvable Jaynes-Cummings (JC) model [5].

Schrödinger cat states [6], the superposition of two coherent states with large amplitudes, have achieved great theoretical and experimental attention, since they are believed to approach semi-classical regions. The properties of such states have been studied in

great detail, see *e.g.* [7]. Entangled coherent states [8] are the generalization of cat states for several modes. These poses the non-classical feature, apart from the superposition, of entanglement. Especially the states of two modes have been investigated [9] and their properties, for example, entanglement [8, 10], decoherence [11, 12] (see also [13]) and non-classical effects [14]. One of the reason for the extensive interest of these states are for their applicability within quantum information processing, see the review article [12] and references therein. For example, quantum teleportation [11, 15], quantum computing [16], non-locality tests [17] and quantum communication [18]. Suggested preparation schemes of entangled coherent states have been covered in a large amount of articles [12]. The proposed methods can be divided into: free propagating waves and linear optical elements [19], Kerr non-linearities [20], trapped ions [21] and cavity QED [22, 23, 24, 25, 26].

The intent of this paper is to present a method, achievable in todays experiments [27, 28, 29], for the generation of entangled coherent states within cavity QED. The model I study here is an extended JC model with a two-level atom interacting with two cavity modes [30, 31, 32, 33]. In the case of modes with equal frequencies, the model can be transformed into the standard JC model by a unitary operation. Thus the system is easily solvable in the new transformed basis. The relation between the standard bare state basis and the new quasi bare state basis is not trivial for any fields. However, we show that for coherent and squeezed state bases a simple transformation relates the two, which are key results of the paper. It is analytically shown how entangled coherent states can be generated in the zero and large atom-field detuning cases. This is done by the knowledge of how the standard JC model evolves for coherent field states [2, 34]. One advantage with the zero detuning scheme is that no atomic measurement is needed, while in the large detuning case a measurement is demanded, which, however, always leaves the field in either of two orthogonal entangled coherent states. Two known approximations are used in deriving the dynamics; the large amplitude approximation, and adiabatic elimination of one of the atomic levels. The former has, to my knowledge, not been considered for the model used in this paper. The latter, adiabatic elimination approximation, is considered in [22], also for the preparation of entangled coherent states. The effective Hamiltonian used in [22] is not the one obtained if the elimination is carried out correctly. The 'false' Hamiltonian does, however, give similar results, and it is explained why this comes about. I give the correct Hamiltonian and discuss how this may generate entangled coherent states, also in the case of different mode frequencies. In [32], analytical expressions are derived for the atomic inversion and phase space distributions, but as a state preparation process it is not discussed. Further, in [33], the same model is used for homodyne detection of the cavity field.

The outline of the paper is: In section 2 the model is introduced and the unitary transformation giving the standard JC model, and then I derive the relations between the original and the quasi modes for coherent and squeezed states. Section 3 considers the zero detuning situation and briefly reviews the dynamics of the standard JC model in the presence of coherent states with large amplitudes. I then show how, within these approximations, the evolution of the combined system may lead to entangled coherent states of the two modes. The following section 4 uses the effective Hamiltonian obtained by adiabatic elimination of one of the two atomic states due to a large atom-field detuning. This Hamiltonian governs an evolution that also results in entangled coherent states. Finally I conclude the paper in section 5 with some comments.

## 2 Describing the model

The JC model [5] has served as a theoretical description of the interaction between a single atom and a single mode inside a high  $Q$  cavity. In these situations only one transition of the atom may be considered, reducing the atom degrees of freedom to a two-level system. The analytically solvable JC model defines a two-level system (spin 1/2) coupled to a harmonic oscillator, within the dipole and rotating wave approximation. In this paper I consider the extended Jaynes-Cummings model, including one two-level atom and two cavity modes, 1 and 2. The Hamiltonian, after adding a second cavity mode, reads ( $\hbar = 1$ )

$$\tilde{H} = \frac{\Omega}{2}\sigma_z + \omega_1 a^\dagger a + \omega_2 b^\dagger b + (g_1 a + g_2 b)\sigma^+ + (g_1^* a^\dagger + g_2^* b^\dagger)\sigma^- . \quad (1)$$

Here the sigma-operators act on the atomic ground and excited states  $|-\rangle$  and  $|+\rangle$ :  $\sigma_z = |+\rangle\langle+| - |-\rangle\langle-|$ ,  $\sigma^+ = |+\rangle\langle-|$  and  $\sigma^- = |-\rangle\langle+|$ . The two operators  $a$  ( $a^\dagger$ ) and  $b$  ( $b^\dagger$ ) are regular boson operators acting on mode 1 and 2 respectively:  $[a, a^\dagger] = 1 = [b, b^\dagger]$  and  $[a, b] = [a^\dagger, b] = \dots = 0$ . The two couplings are given by  $g_{1,2}$ , the mode frequencies by  $\omega_{1,2}$  and the atomic transition frequency by  $\Omega$ . Bare states are given by  $|n\rangle_1|m\rangle_2|\pm\rangle$ , where the first two kets give the number of photons in mode 1 and 2 respectively,  $a^\dagger a|n\rangle|m\rangle|\pm\rangle = n|n\rangle|m\rangle|\pm\rangle$  and  $b^\dagger b|n\rangle|m\rangle|\pm\rangle = m|n\rangle|m\rangle|\pm\rangle$ , while the last one gives the atomic state,  $\sigma_z|n\rangle|m\rangle|\pm\rangle = \pm|n\rangle|m\rangle|\pm\rangle$ . The number of excitations are preserved by the Hamiltonian (1);  $[N, H] = 0$ , where  $N = \frac{1}{2}\sigma_z + a^\dagger a + b^\dagger b$ . It is convenient to work in the interaction picture with respect to the operator  $N$ , and from now on we assume  $\omega_1 = \omega_2 = \omega$  and the couplings  $g_{1,2}$  to be real, giving the interaction picture Hamiltonian

$$H = \tilde{H} - \omega N = \frac{\Delta}{2}\sigma_z + (g_1 a + g_2 b)\sigma^+ + (g_1 a^\dagger + g_2 b^\dagger)\sigma^- , \quad (2)$$

where  $\Delta = \Omega - \omega$  is the atom-field detuning. In order to proceed I introduce the new boson operators  $A$  and  $B$ , acting on quasi modes  $I$  and  $II$  respectively, by a orthogonal transformation  $U$  according to

$$\begin{bmatrix} A \\ B \end{bmatrix} = U \begin{bmatrix} a \\ b \end{bmatrix} , \quad U = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} , \quad (3)$$

where  $\cos(\theta) = g_1/g$  and  $g = \sqrt{g_1^2 + g_2^2}$ . With the boson operators  $A$  and  $B$ , the Hamiltonian has the form of a standard JC one for quasi mode  $I$ ,

$$H = \frac{\Delta}{2}\sigma_z + g(A^\dagger\sigma^- + A\sigma^+). \quad (4)$$

Note that  $a^\dagger a + b^\dagger b = A^\dagger A + B^\dagger B$ , meaning that  $\Delta$  is also the detuning between the atom and quasi fields  $I$  and  $II$ . The number, or Fock, states for the quasi modes are written as  $|N\rangle_I$  and  $|M\rangle_{II}$ , and it follows that quasi mode  $II$  is unaffected by the Hamiltonian  $H$  in the interaction picture. While in the non interaction pictures it accumulate a phase proportional to  $\omega B^\dagger B$ . In the zero detuning case, an initial state

$$|\Psi(0)\rangle = \sum_{N,M} C_N^{(I)} C_M^{(II)} |N\rangle_I |M\rangle_{II} [\gamma|-\rangle + \delta|+\rangle] \quad (5)$$

evolves into

$$\begin{aligned}
|\Psi(t)\rangle &= \left[ \sum_M C_M^{(II)} |M\rangle_{II} \right] \frac{1}{\sqrt{2}} \sum_N \left[ \left( \gamma C_{N+1}^{(I)} + \right. \right. \\
&\quad \left. \left. + \delta C_N^{(I)} \right) e^{igt\sqrt{N+1}} |\phi_{1N}\rangle_I \right. \\
&\quad \left. + \left( \delta C_N^{(I)} - \gamma C_{N+1}^{(I)} \right) e^{-igt\sqrt{N+1}} |\phi_{2N}\rangle_I \right], \tag{6}
\end{aligned}$$

where we have introduced the quasi dressed states

$$\begin{aligned}
|\phi_{1N}\rangle_I &= \frac{1}{\sqrt{2}} [ |N\rangle_I |+\rangle + |N+1\rangle_I |-\rangle ] \\
|\phi_{2N}\rangle_I &= \frac{1}{\sqrt{2}} [ |N\rangle_I |+\rangle - |N+1\rangle_I |-\rangle ]. \tag{7}
\end{aligned}$$

The particular case of single quasi Fock states,  $C_N^{(I)} = \delta_{NN'}$ , has been studied in [30], and here I consider coherent or squeezed states. The displacement operator  $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ , creates a coherent state with amplitude  $\alpha$  when it operates on vacuum,  $D(\alpha)|0\rangle = |\alpha\rangle$ . Likewise, the operators  $A$  and  $B$  may be used to define displacement operators for quasi modes  $I$  and  $II$ . If both modes 1 and 2 are prepared in coherent states, it is easy to find corresponding states of the quasi modes  $I$  and  $II$  using eq (3),

$$\begin{aligned}
D_1(\alpha)D_2(\beta) &= \exp(\alpha a^\dagger - \alpha^* a + \beta b^\dagger - \beta^* b) \\
&= \exp[\alpha(\cos(\theta)A^\dagger - \sin(\theta)B^\dagger) \\
&\quad - \alpha^*(\cos(\theta)A - \sin(\theta)B) \\
&\quad + \beta(\sin(\theta)A^\dagger + \cos(\theta)B^\dagger) \\
&\quad - \beta^*(\sin(\theta)A + \cos(\theta)B)] \\
&= \exp[(\alpha \cos(\theta) + \beta \sin(\theta)) A^\dagger \\
&\quad - (\alpha^* \cos(\theta) + \beta^* \sin(\theta)) A \\
&\quad + (\beta \cos(\theta) - \alpha \sin(\theta)) B^\dagger \\
&\quad - (\beta^* \cos(\theta) - \alpha^* \sin(\theta)) B] \\
&= D_I(\mu)D_{II}(\nu), \tag{8}
\end{aligned}$$

where

$$\begin{bmatrix} \mu \\ \nu \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \tag{9}$$

Thus, the amplitudes  $\alpha$ ,  $\beta$ ,  $\mu$  and  $\nu$  are related in the same way as the boson operators  $a$ ,  $b$ ,  $A$  and  $B$ . It is understood that the relation (8) works both ways,  $1, 2 \leftrightarrow I, II$ . A special case of this relation between the basis is mentioned in [33]

Let us now derive the corresponding relation between the squeezing operators  $S(z) = \exp[\frac{1}{2}(z^*a^2 - za^{\dagger 2})]$ , see [35], for the various modes. Using the Campbell-Baker-Hausdorff

theorem [35],  $\exp(A + B) = \exp(A)\exp(B)\exp(-[A, B]/2)$ , where the operators  $A$  and  $B$  fulfills  $[A, [A, B]] = 0 = [B, [A, B]]$ , it follows

$$S_1(z_1)S_2(z_2) = e^{(p^*AB - pA^\dagger B^\dagger)}S_I(q)S_{II}(q), \quad (10)$$

where  $p = \sin(2\theta)(z_2 - z_1)/2$  and  $q = z_1 \cos^2(\theta) + z_2 \sin^2(\theta)$ . When  $z_1 = z = z_2$ ,  $p = 0$  and  $q = z$ , and the first exponential on the right hand side disappears, resulting in two ordinary squeezing operators for mode  $I$  and  $II$ .

The results presented in eqs. (8), (9) and (10) are a main underlying condition for this paper; an entangled coherent or Schrödinger cat state in one basis will give a similar state in the other basis. This might be an intuitive result, but one should remember that the unitary transformation (3) for making the 2-mode JC model (1) into a standard JC model, has the simple form since the two modes have identical frequencies. In section 4 for large detunings, it will be shown that the generation of entangled coherent states for modes with different frequencies is more complicated.

### 3 Generation of entangled states 1; zero detuning

In the previous section it was shown that having two coherent states in modes 1 and 2 is the same as having two coherent states in modes  $I$  and  $II$  with different amplitudes. The dynamics of the original JC model in the case of a coherent state with a large amplitude is well understood [27, 34], see also [37, 38]. Surprisingly, in the large amplitude limit it was found that, independently of the atoms initial state it disentangles from the field after a particular time, the half revival time  $t_R = 2\pi\sqrt{\bar{n}}/g$  where  $\bar{n}$  is the average photon number. At this time the initial coherent state has split up into a superposition of two coherent states with opposite phases, a so called Schrödinger cat state. I briefly review this phenomenon here and for a more rigorous mathematical treatment see [34].

For initial coherent states we have

$$C_{N+1} = e^{-i\varphi} \left( \frac{\bar{N}}{N+1} \right)^{1/2} C_N, \quad (11)$$

where  $\varphi$  is the phase of the field;  $\mu = \sqrt{N}e^{-i\varphi}$ . In the limit of large  $\bar{N}$ , it is legitimate to approximate  $C_{N+1} \approx e^{-i\varphi}C_N$ . Since, for the coherent state with large amplitude, the distribution  $C_N$  is sharply peaked around its average  $\bar{N}$ , I expand the exponent accordingly

$$\begin{aligned} \sqrt{N+1} &\approx \sqrt{N} + \frac{1}{2\sqrt{N}} \approx \sqrt{N} + \frac{1}{2\sqrt{N}}, \\ \sqrt{N} &\approx \frac{\sqrt{N}}{2} + \frac{N}{2\sqrt{N}} - \frac{(N-\bar{N})^2}{8N^{3/2}} \dots \end{aligned} \quad (12)$$

For simplicity, assume  $\gamma = 1$ , then the time-dependent solution (6), using the above approximations, reduces to

$$\begin{aligned} |\Psi(t)\rangle &\approx \left[ \sum_M C_M^{(II)} |M\rangle_{II} \right] \frac{1}{2} \left[ e^{igt\sqrt{N}/2} \sum_N C_N^{(I)} \times \right. \\ &\quad e^{igtN/2\sqrt{N}} |N\rangle_I \left( e^{igt/2\sqrt{N}} |+\rangle + e^{i\varphi} |-\rangle \right) \\ &\quad \left. + e^{-igt\sqrt{N}/2} \sum_N C_N^{(I)} e^{-igtN/2\sqrt{N}} |N\rangle_I \times \right. \\ &\quad \left. \left( e^{-igt/2\sqrt{N}} |+\rangle - e^{i\varphi} |-\rangle \right) \right] \end{aligned} \quad (13)$$

Now, at  $t = t_R/2$  it is clear that the atomic and field states separates, and for a coherent distribution in quasi mode  $I$  with amplitude  $\mu$ , the state becomes

$$\begin{aligned} |\Psi(t=t_R/2)\rangle &= \left[ \sum_M C_M^{(II)} |M\rangle_{II} \right] \frac{1}{\sqrt{2}} (i|+\rangle + e^{i\varphi} |-\rangle) \times \\ &\quad \frac{1}{\sqrt{2}} (e^{i\pi\bar{N}} |i\mu\rangle_I - e^{-i\pi\bar{N}} | -i\mu\rangle_I), \end{aligned} \quad (14)$$

where we assume  $\langle -i\mu | i\mu \rangle \approx 0$  in the large amplitude limit. Thus, within the validity of the approximations, as the atom disentangle from the field, the field has split up into a superposition of coherent states with opposite phases. The interesting fact is that the disentanglement appears for any initial atomic state, the similar calculation as in eq. (13) could be performed for  $\delta = 1$  and since  $|\phi_{1,2}\rangle$  span the whole atomic subspace, it holds for any initial state. By using squeezed states which may be more peaked around their means  $\bar{N}$ , the validity constrains are more easily fulfilled [25, 37]. The similar splitting of the field state in phase space is present for a squeezed state.

Going back to our extended JC model with two modes, we assume that modes 1 and 2 are initially in coherent states with large amplitudes  $\alpha$  and  $\beta$ . The quasi modes  $I$  and  $II$  will then be in coherent states with amplitudes  $\mu$  and  $\nu$  according to eq. (9). By letting the atom interact for a time  $t = t_R/2$ , where  $t_R$  is the revival time for the quasi mode  $I$ ,  $t_R = 2\pi\sqrt{N}/g$ , the field becomes

$$|\psi_f(t_R/2)\rangle = \frac{1}{\sqrt{2}} (e^{i\pi\bar{N}} |i\mu\rangle_I - e^{-i\pi\bar{N}} | -i\mu\rangle_I) |\nu\rangle_{II}. \quad (15)$$

Going to modes 1 and 2 we get

$$|\psi_f(t_R/2)\rangle = \frac{1}{\sqrt{2}} (e^{i\pi\bar{N}} |\alpha'\rangle_1 |\beta'\rangle_2 - e^{-i\pi\bar{N}} |\alpha''\rangle_1 |\beta''\rangle_2), \quad (16)$$

where

$$\begin{aligned} \begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} &= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \\ \begin{bmatrix} \alpha'' \\ \beta'' \end{bmatrix} &= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} -i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \end{aligned} \quad (17)$$

For  $\alpha = -i\sqrt{n} = \beta$  and  $g_1 = g_2$ , the angle  $\theta$  is  $\pi/2$ , giving

$$|\psi_f(t_R/2)\rangle = \frac{1}{\sqrt{2}} (e^{i2\pi\bar{n}} |\bar{n}\rangle_1 |\bar{n}\rangle_2 - e^{-i2\pi\bar{n}} | -\bar{n}\rangle_1 | -\bar{n}\rangle_2). \quad (18)$$

## 4 Generation of entangled states 2; large detuning

When the atom-field detuning is large compared to the coupling, population is unlikely to transfer between the internal atomic states  $|\pm\rangle$ . It is then possible to approximate the JC model by an effective one where the initially empty level has been adiabatically eliminated. In the appendix A I derive an effective JC model by transforming the Hamiltonian by an unitary operator [39, 40]. The resulting Hamiltonian, to order  $\mathcal{O}(g^2/\Delta)$ , becomes

$$H_{el} = \left[ \frac{\Delta}{2} + \frac{g^2}{\Delta} A^\dagger A \right] \sigma_z + \frac{g^2}{2\Delta} \sigma^+ \sigma^-. \quad (19)$$

Evolving an initial state (5) by this Hamiltonian gives

$$\begin{aligned}
|\Psi(t)\rangle &= U(t)|\Psi(0)\rangle = \left[ \sum_M C_M^{(II)} |M\rangle_{II} \right] \times \\
&\times \sum_N \left[ C_N^{(I)} \gamma e^{-i\left(\frac{\Delta}{2} + \frac{g^2 N}{\Delta}\right)t} |N\rangle_I |-\rangle \right. \\
&\left. + \delta e^{+i\left(\frac{\Delta}{2} + \frac{g^2}{2\Delta} + \frac{g^2 N}{\Delta}\right)t} |N\rangle_I |+\rangle \right]. \tag{20}
\end{aligned}$$

And for initial coherent states with amplitudes  $\mu$  and  $\nu$  we get

$$|\Psi(t)\rangle = \left[ \gamma |\mu e^{-i\frac{g^2 t}{\Delta}}\rangle_I |-\rangle + \delta e^{i\left(\Delta + \frac{g^2}{2\Delta}\right)t} |\mu e^{i\frac{g^2 t}{\Delta}}\rangle_I |+\rangle \right] |\nu\rangle_{II}. \tag{21}$$

Assume  $\gamma = \delta = 1/\sqrt{2}$  and that the atom is measured at time  $t'$  in the  $\frac{1}{\sqrt{2}}(|-\rangle \pm |+)\rangle$  basis, leaving the fields in the states

$$\begin{aligned}
|\psi_f(t')\rangle &= \frac{1}{\sqrt{2}} \left[ |\mu e^{-i\frac{g^2 t'}{\Delta}}\rangle_I |\nu\rangle_{II} + e^{i\left(\Delta + \frac{g^2}{2\Delta}\right)t'} |\mu e^{i\frac{g^2 t'}{\Delta}}\rangle_I |\nu\rangle_{II} \right], \\
|\psi_f(t')\rangle &= \frac{1}{\sqrt{2}} \left[ |\alpha'\rangle_1 |\beta'\rangle_2 + e^{i\left(\Delta + \frac{g^2}{2\Delta}\right)t'} |\alpha''\rangle_1 |\beta''\rangle_2 \right], \tag{22}
\end{aligned}$$

where the amplitudes  $\alpha'$ ,  $\beta'$ ,  $\alpha''$  and  $\beta''$  are given by eq. (17) with the middle matrix changed to

$$\begin{bmatrix} e^{-i\frac{g^2 t'}{\Delta}} & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} e^{i\frac{g^2 t'}{\Delta}} & 0 \\ 0 & 1 \end{bmatrix}, \tag{23}$$

respectively.

In [22], the same method as the one in this section is suggested for entangled coherent state preparation. They start with an identical Hamiltonian, but without the restriction on equal frequencies of the two modes, and perform an adiabatic elimination, in the original basis, giving the interaction Hamiltonian

$$H_I = \frac{g_1^2}{\Delta_1} a^\dagger a \sigma_z + \frac{g_2^2}{\Delta_2} b^\dagger b \sigma_z, \tag{24}$$

where  $\Delta_i = \Omega - \omega_i$ ,  $i = 1, 2$ . The interesting observation is that the cross terms  $a^\dagger b$  and  $ab^\dagger$  are omitted in (24), with the conclusion that the elimination has not been carried out correctly. Still, both Hamiltonians (19) and (24) generate entangled coherent states, which is a consequence of the relation (8). This indicates the importance of this result; in spite its simplicity, it is nontrivial and a direct consequence from the properties of coherent states. The correct expression for the interaction picture Hamiltonian, after the adiabatic elimination has been performed (analogous to the procedure presented in the

appendix A), reads

$$\begin{aligned}
H_I = & \frac{g_1^2}{\Delta_1} a^\dagger a \sigma_z + \frac{g_2^2}{\Delta_2} b^\dagger b \sigma_z \\
& + \frac{1}{2} \left( \frac{g_1 g_2}{\Delta_1} + \frac{g_1 g_2}{\Delta_2} \right) (a^\dagger b + a b^\dagger) \sigma_z \\
& + \frac{1}{2} \left( \frac{g_1^2}{\Delta_1} + \frac{g_2^2}{\Delta_2} \right) \sigma^+ \sigma^-,
\end{aligned} \tag{25}$$

which reproduces the effective Hamiltonian (19) by letting  $g_2 = 0$ . This is a Hamiltonian for two coupled oscillators, which may be decoupled by introducing the boson operators [41]

$$\tilde{A} = a \cos(\eta) + b \sin(\eta), \quad \tilde{B} = -a \sin(\eta) + b \cos(\eta), \tag{26}$$

giving

$$H_I = \left( \lambda \tilde{A}^\dagger \tilde{A} + \zeta \tilde{B}^\dagger \tilde{B} \right) \sigma_z + \frac{1}{2} (\lambda + \zeta) \sigma^+ \sigma^-. \tag{27}$$

The parameters are expressed in the old ones as,

$$\begin{aligned}
\lambda &= \frac{1}{2} \left[ \frac{g_1^2}{\Delta_1} (1 + \cos(2\eta)) + \frac{g_2^2}{\Delta_2} (1 - \cos(2\eta)) \right], \\
\zeta &= \frac{1}{2} \left[ \frac{g_1^2}{\Delta_1} (1 - \cos(2\eta)) + \frac{g_2^2}{\Delta_2} (1 + \cos(2\eta)) \right],
\end{aligned} \tag{28}$$

with

$$\eta = \frac{1}{2} \tan^{-1} \frac{g_1 g_2 \Delta_2 + g_1 g_2 \Delta_1}{g_1^2 \Delta_2 - g_2^2 \Delta_1}. \tag{29}$$

Thus, the equations (27)-(29) solves the more general problem of large, but different, detunings.

## 5 Conclusion

In this paper I have presented a simple model for generation of highly non-classical entangled coherent states. A two-level atom interacts with two cavity modes with equal frequencies, which under a unitary transformation becomes identical to the standard JC model describing the interaction between one two-level atom and one cavity mode. The known analytical results of the original JC model are used to derive the time-evolution of this extended system. The special situation of initial coherent states in the two modes are considered, and it was shown how these may evolve into entangled coherent states of the modes. Both the zero and large atom-field detuning situations are studied and the large amplitude approximations are used in the former. An advantage with a vanishing detuning is that characteristic interaction times usually are shorter, giving smaller losses. However, there is a conflict; for larger amplitudes of the initial coherent states the approximations are better fulfilled, while the decoherence times are decreased. The validity of these approximations and decoherences have been studied in the literature [27, 25, 37]. The opposite limit of large detuning was already studied in [22] for generation of entangled coherent states. They, however, used an effective model that omitted two essential terms, which will result in new coupling parameters and new phases of the involved coherent states.

Note that if the amplitudes of the two modes 1 and 2 are equal,  $\alpha = \beta$ , and they couple with the same strength to the atom,  $g_1 = g_2$ , we find  $\mu = \sqrt{2}\alpha$  meaning that the large amplitude approximation works better for quasi mode  $I$  than for the individual modes 1 and 2 separately. The decay rate of the decoherence terms of the state  $\rho$  of a 1-mode Schrödinger cat or a 2-mode entangled coherent state are proportional to  $|\alpha|^2$  or  $|\alpha|^2 + |\beta|^2$  respectively [12, 42], where  $\alpha$  and  $\beta$  are the amplitudes of the two modes. Thus, they scale the same as for the situation with  $\mu = \sqrt{2}\alpha$  above, indicating that the model is not more sensitive to decoherence as could be expected from the two modes included. Schrödinger cat states have been prepared experimentally, both within the large amplitude and the large detuning approximations, see [27, 28, 29]. Hence, the experimental verification of entangled coherent states should be possible within current setups. In addition, the method presented is as simple as possible; the degrees of freedom is minimized (no external fields or atomic levels) and as few experimental steps as possible, which should maximize the preparation fidelity. However, the measurement process of entangled coherent states, not discussed in this paper, turns out to be nontrivial [43].

The advantage of the large detuning case is that the amplitudes of the coherent states, need not be large for the scheme to work. The model presented in this paper can be generalized to work for preparing  $l$  mode entangled coherent states. Then the transformation (3) is extended to contain all the  $l$  boson operators.

In the model, the two-level atom couples to two cavity modes, so that the modes must have the same polarizations due to selection rules. One interesting situation where this is achievable is by using overlapping cavities [26]. The atom interacts with the two modes, belonging to different cavities, in a region where the cavity fields overlap in space. This has the additional interesting aspect of entangled states spatially separated. In [24] a scheme for preparation of entangled coherent states of separated cavities are presented, by a two-level atom interacting in succession with the cavities. The advantage with such a model is that the fields does not need to overlap, while the disadvantage is that the interaction time increases and decoherence effects of the atom during the flight between the cavities may be of importance.

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## A Adiabatic elimination

The interaction picture JC Hamiltonian is

$$H = \frac{\Delta}{2}\sigma_z + g(a^\dagger\sigma^- + a\sigma^+). \quad (30)$$

Following [39] I introduce the unitary transformation

$$U = e^S, \quad (31)$$

where

$$S = \lambda(a\sigma^+ - a^\dagger\sigma^-), \quad (32)$$

for some constant  $\lambda$  to be determined later. Using the operator formula

$$X' = e^S X e^{-S} = X + [S, X] + (1/2!)[S, [S, X]] + \dots, \quad (33)$$

and keeping terms to second order in the coupling one obtains

$$\begin{aligned}
a' &= a + \lambda\sigma^-, \\
\sigma^- &= \sigma^- + \lambda a\sigma_z, \\
\sigma_z &= \sigma_z - 2\lambda(a^\dagger\sigma^- + a\sigma^+) - 2\lambda^2 a^\dagger a\sigma_z - \lambda^2\sigma^+\sigma^-.
\end{aligned} \tag{34}$$

By choosing  $\lambda = g/\Delta$  the transformed Hamiltonian, to order  $\mathcal{O}(g^2/\Delta)$ , becomes

$$H' = \frac{\Delta}{2}\sigma_z + \frac{g^2}{\Delta}a^\dagger a\sigma_z + \frac{g^2}{2\Delta}\sigma^+\sigma^-.\tag{35}$$

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